Constrained Dispatch

Description

This document describes Aurora's mathematical formulation of the constrained dispatch problem. The constrained dispatch problem is used to determine the optimal generation pattern and flow between zones, subject to monthly and annual constraints including resource emission, energy, and fuel production. The solution takes into account the standard dispatch constraints including transmission flow limits, resource generating limits, and a demand requirement. Note that while this documentation will refer in general to annual constraints, the emission, energy, and fuel constraints may also be applied to any subset of months in a year.

Introduction

The model seeks to minimize total cost on the system subject to the annual constraints. The constraint logic groups together similar dispatch hours for each month into "buckets" in order to create a manageable problem size. The number of buckets is determined by the Constraint Segment Definition. For example, Medium precision creates 24 segments for the month: one bucket for all Hour 1's, another for all Hour 2's, etc. Using these data buckets, the model solves a linear program (LP) that takes into account standard dispatch constraints (transmission flow limits, resource generating limits, and demand requirements) and annual constraints (emission, energy, and fuel). Decision variables for each bucket of the LP are resource dispatch and transmission between zones.

Finally, the model passes information from the LP solution to the chronological dispatch which alters the resource operation in order to honor the annual targets. This pass is made using one of two methods: Hourly Limit or Shadow Adder. The Hourly Limit method (default) uses a constraint in the chronological LP dispatch to limit the operation of the resources to match the allocation in the LDC solution. In general, the Hourly Limit approach will result in an annual chronological solution that adheres more closely to the input limit. When the Shadow Adder method is used, the model calculates a $/MWh shadow price adder that is used to adjust the dispatch cost of each resource in the constraint. This approach typically works well for constraints with a large number of participating resources.

Mathematical Framework

Resolving Data

Data is resolved for all dispatch hours in the period and grouped into buckets by demand similarity. All bucket data (demand, capacity, etc.) is averaged. The user may specify the number of buckets by setting the constraint segment definition precision. These buckets then become periods for the annual LP.

Notation

Let $R_1, R_2, ..., R_n$ be a given set of resource segments, and let $Z_1, Z_2, ..., Z_m$ be a given set of zones. Note that each resource must belong to exactly one zone. Also note that in the mathematical notation below (with the exception of subscripts and indices), all lower-case letters represent decision variables and all upper-case letters represent data input by the user or derived beforehand by the model. Let M be the set of all minimum resource segments, and let H be the set of all dispatchable (non-minimum) resource segments. Let B be the number of periods with $p \in \{1, ..., B\}$ being an index denoting the period. Define the following:

r_i^p = dispatch for resource segment R_i in period p (decision variables)
t_{ij}^p = transmission from Z_i to Z_j in period p (decision variables)
U_i^p = average capability of resource segment R_i in period p
C_i^p = average dispatch cost of resource segment R_i in period p
T_{ij}^p = average transmission capacity from Z_i to Z_j in period p
L_{ij}^p = average loss factor for transmission from Z_i to Z_j in period p
W_{ij}^p = average wheeling charge for transmission from Z_i to Z_j in period p
D_j^p = average demand for zone Z_j in period p

Formulation of the LP

The model creates the following objective function by finding the total cost on the system. Total cost is the sum of total resource costs and total wheeling costs, summed over all periods:

$$\sum_{p=1}^{B}\left(\sum_{i=1}^{n} r_i^p C_i^p + \sum_{i=1}^{m}\sum_{j=1}^{m} t_{ij}^p W_{ij}^p\right) \tag{1}$$

Energy constraints ensure that minimum and maximum energy requirements are obeyed. That is, for all $i \in \{1, 2, ..., n\}$:

$$0 \leq r_i^p \leq U_i^p \tag{2}$$

Transmission constraints require that transmissions between zones are within capacity for each period. For all $i, j \in \{1, 2, ..., m\}$ (with $T_{ij} = 0$ for $i = j$) and for all $p \in \{1, ..., B\}$:

$$0 \leq t_{ij}^p \leq T_{ij}^p \tag{3}$$

Demand constraints require that total supply equals total demand in each zone for all periods. For all $j \in \{1, 2, ..., m\}$ and for all $p \in \{1, ..., B\}$:

$$D_j^p = \sum_{R_i \in Z_j} r_i^p + \sum_{i=1}^{m}(L_{ij}^p t_{ij}^p - t_{ji}^p) \tag{4}$$

Because the commitment status is not known beforehand, minimum segment constraints ensure that the LP does not dispatch a commitment resource's non-minimum segment without also dispatching the minimum segment. Let R_m^p denote a commitment resource's minimum segment in period p. Let N be the set of the resource's dispatchable segments in period p. For each commitment resource the following must hold for all p:

$$\frac{r_m^p}{U_m^p} \geq \frac{\sum_{R_i^p \in N} r_i^p}{\sum_{R_i^p \in N} U_i^p} \tag{5}$$

The objective function coefficient for each minimum segment variable is also adjusted to give a representation of the start costs (if applicable).

Constraint Types

The user may impose the annual constraints listed below on the model. Note again that while this documentation treats the constraints as annual restrictions, they may also be applied monthly or over a subset of months. These constraints may include any of the following:

Emission: This limits the total annual emission of a set S of resources to an amount X tons. Let E_i^p be the emission rate in tons/MWh of resource segment R_i in period p.

$$\sum_{p=1}^{B} \sum_{R_i \in S} E_i^p r_i^p \leq X \tag{6}$$

Emission Rate: This limits the annual emission rate of a set S of resources to X tons/MWh. Let E_i be the emission rate in tons/MWh of resource segment R_i in period p.

$$\frac{\sum_{p=1}^{B} \sum_{R_i \in S} E_i^p r_i^p}{\sum_{p=1}^{B} \sum_{R_i \in S} r_i^p} \leq X \tag{7}$$

If the Emission Rate Group input is employed, then each pair of constraints in the group is linked with an additional decision variable in the denominator representing the MWh traded between groups.

Energy Max: This limits the total energy production of a set S of resources to an amount no greater than X MWh over the year.

$$\sum_{p=1}^{B} \sum_{R_i \in S} r_i^p \leq X \tag{8}$$

Energy CF Max: This limits the annual capacity factor for a set S of resources to a value no greater than X. Let V_i^p be the capacity in MWh of resource segment R_i in period p.

$$\frac{\sum_{p=1}^{B} \sum_{R_i \in S} r_i^p}{\sum_{p=1}^{B} \sum_{R_i \in S} V_i^p} \leq X \tag{9}$$

Energy Min: This requires that the total annual energy production of a set S of resources be no less than X MWh.

$$\sum_{p=1}^{B} \sum_{R_i \in S} r_i^p \geq X \tag{10}$$

Energy CF Min: This requires that the annual capacity factor of a set S of resources be no less than a value X. Let V_i^p be the capacity in MWh of resource segment R_i in period p.

$$\frac{\sum_{p=1}^{B} \sum_{R_i \in S} r_i^p}{\sum_{p=1}^{B} \sum_{R_i \in S} V_i^p} \geq X \tag{11}$$

Fuel: This places an annual limit X MMBtu on the total fuel usage of the set S of all resources that use the specified fuel. Let Q_i^p denote the heat rate of resource segment R_i in Btu/kWh in period p.

$$\sum_{p=1}^{B} \sum_{R_i \in S} \frac{Q_i^p r_i^p}{1000} \leq X \tag{12}$$

To summarize, the model formulates the following LP and finds the vectors $\boldsymbol{r^p}$ and $\boldsymbol{t^p}$ of resource dispatch and transmission between zones, respectively, for all $p \in \{1, ..., B\}$. The optimization is subject to the user's choice of annual constraints detailed above, as well as the global constraints:

$$
\begin{array}{lll}
\text{minimize} & \sum_{p=1}^{B}\left(\sum_{i=1}^{n} r_i^p C_i^p + \sum_{i=1}^{m}\sum_{j=1}^{m} t_{ij}^p W_{ij}^p \right) & \\
\text{subject to} & 0 \leq r_i^p \leq U_i^p & \text{for all } i \text{ and all } p \\
 & 0 \leq t_{ij}^p \leq T_{ij}^p & \text{for all } i \neq j \text{ and all } p \\
 & D_j^p = \sum_{R_i \in Z_j} r_i^p + \sum_{i=1}^{m}(L_{ij}^p t_{ij}^p - t_{ji}^p) & \text{for all } j \text{ and all } p \\
 & \frac{r_m^p}{U_m^p} \geq \frac{\sum_{R_i^p \in N} r_i^p}{\sum_{R_i^p \in N} U_i^p} & \text{for all commitment resources and all } p \\
 & \text{User's choice of emission, energy, and fuel constraints} &
\end{array}
$$

